Original Article

On an Application of Geiger–Muller Counter Model (Type-II) for Optimization Relating to Hospital Administration

Suddhendu Biswas, H. D. Arora[1]

Departments of Mathematics and [1]Statistics, Amity Institute of Applied Sciences, Amity University, Noida, Uttar Pradesh, India

Abstract

Assuming that n patients in surgical or maternity wards are admitted T days (T = 0, 1, 2 … n) prior to the date of surgery or delivery for clinical check-up, an attempt has been made to develop the best appropriate probability model on Geiger–Muller counter type-II, to describe the arrival of patients and the busy (locked) period in the hospital. The problem is further extended to obtain optimal solution loading to (a) uniform type of medical care and (b) for providing medical care for a maximum period depending on the complications on an average, subject to the fixed budget of the hospital.

Keywords: Blocks and antiblocks, busy period, Geiger–Muller counter model type-II, surgical and maternity cases

Introduction

Suppose a specific type of surgical or maternity cases are admitted in the hospital prior to T days (T = 0, 1, 2… n) before the date of surgery or delivery. Naturally, it would have been desirable to allow T (on the part of hospital authorities) sufficiently large to present the possible complications associated with surgery or delivery. But as a matter of fact due to severe rush or demand for a bed in the hospital as well as constraints on the hospital budget, the hospital authorities are found to put some ceilings on the upper bound of T.

Afrane and Appah[1] established that queuing theory and modeling is an effective tool that can be used to make decisions on staffing needs for optimal performance with regard to queuing challenges in hospitals. Large number of work has been done in related topics by Biswas,[2] Garwood,[3] Oliver,[4] Adele and Barry,[5] Cochran and Bharti,[6] Davies and Davies,[7] Green,[8] Green *et al*.,[9] Hall *et al*.,[10] Ozcan,[11] Roche *et al*.,[12] Biswas *et al*.,[13] and Singh.[14]

The problem considered in this article is a suitable model representing the arrival of the patients in the hospital as well as their service time with an objective of obtaining an optimal distribution of T for providing (n) a uniform type of medical care as far as practicable as well.

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	DOI: 10.4103/ami.ami_26_17

Development of the Model

Let us consider a time period covering N surgical or maternity cases. Assume the time gap between two consecutive dates of operation or delivery follows a negative exponential distribution with parameter λ (λ > 0). It follows that the expected total time period covered is $\frac{N}{\lambda}$.

A representation of Raff[15] reveals that the total period $\frac{N}{\lambda}$ can be considered as the sum of several blocks and antiblocks from the point of view of hospital administration. Each block consists of a minimum length T corresponding to a fixed observation period T per patient, which naturally gets extended with the arrival of fresh case with the same observation period, while the previous case is still under observation. On the other hand, antiblocks precisely correspond to the period during which the hospital resources are not utilized by the patients, for keeping the patient under observation.

The blocks and the antiblocks thus correspond to the dead (paralyzed) and the free (unlocked) periods of n

Address for correspondence: Prof. H. D. Arora,
Department of Mathematics, Amity Institute of Applied Sciences,
Amity University, Noida - 201 313, Uttar Pradesh, India.
E-mail: hdarora@amity.edu

For reprints contact: reprints@medknow.com

How to cite this article: Biswas S, Arora HD. On an application of geiger–Muller counter model (Type-II) for optimization relating to Hospital Administration. Acta Med Int 2017;4:16-9.

Geiger–Muller counter model type-II (Karlin and Taylor[16]) with fixed dead time. The blocks and antiblocks are shown in Figure 1 for $T = 3$. The date of admission of the patients in the hospital, A_i and B_i (i-1, 2,…) shows the length of the antiblocks and blocks, respectively.

Let P denotes the probability of an event and X (a random variable) be the time gap between two consecutive surgeries or deliveries.

Then, it follows that:

$$P(X > T) = e^{-\lambda T}$$

and $P(X > T + t) = e^{-\lambda(T+t)}$; $t > 0$

or $P(T < X \le T + t) = e^{-\lambda T} - e^{-\lambda(T+t)}$

$$= e^{-\lambda T}(1 - e^{-\lambda t}) \quad (1)$$

But

$$P(T < X \le T + t) = P(\textit{that there is an antiblock of size} \le t)$$

(due to one-to-one correspondence between antiblocks and gaps of size > T).

Therefore, the expected number of antiblocks

$$(\textit{of size} \le t) = Ne^{-\lambda T}\left(1 - e^{-\lambda t}\right)$$

Hence, the expected number of antiblocks of size between $(t,\ t + T)$

$$= N\lambda e^{-\lambda T} e^{-\lambda t} dt \quad (2)$$

The average duration of time spent in the antiblocks:

$$= Ne^{-\lambda T}\lambda \int_0^\infty t e^{-\lambda t} dt$$

$$= \frac{Ne^{-\lambda T}}{\lambda} \quad (3)$$

Therefore, the average time spent on all the blocks:

$$= \left[\frac{N}{\lambda} - \frac{N}{\lambda} e^{-\lambda T}\right]$$

$$= \frac{N}{\lambda}(1 - e^{-\lambda T}) \quad (4)$$

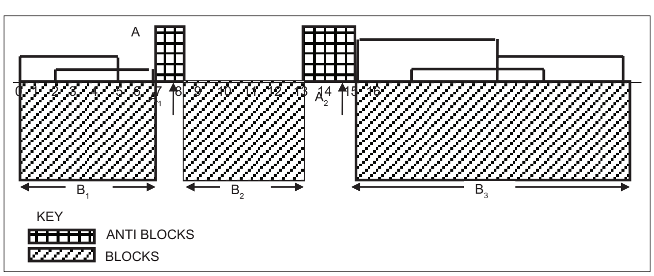

Figure 1: The blocks and antiblocks for $T = 3$

Noting that the expected time spent on blocks has been utilized by N patients, the average time spent per patient is equal to

$$\frac{1}{\lambda}(1 - e^{-\lambda T}) \quad (5)$$

Next assuming that T is a random variable with binomial distribution with parameters n and p.

$$P(T = r) = \binom{n}{r} p^r (1 - p)^{n-r}; r = 0,1,2, \quad (6)$$

Now the expected time taken per patient has been derived in (7) and the total expected cost per day accordingly is given in (8)

$$= \sum_{r=0}^{n} \frac{(1 - e^{-\lambda r})}{\lambda} P\ (T = r)$$

$$= \frac{\sum_{r=0}^{n}(1 - e^{-\lambda r})}{\lambda} \binom{n}{r} p^r (1 - p)^{n-r}$$

$$= \frac{1}{\lambda}\left[1 - \left\{1 - p(1 - e^{-\lambda})\right\}^n\right]$$

Therefore, the total expected cost per day:

$$= \frac{N\pi}{\lambda}\left[1 - \left\{1 - p(1 - e^{-\lambda})\right\}^n\right] \quad (8)$$

Where π stands for the cost per day.

Optimization

Uniform type of medical care

One of the items of investigation is to obtain a condition for providing a uniform type of medical care to all the cases under observation subject to the available budget of the hospital. More precisely, the purpose is to obtain the estimates of the parameters of the distribution of T such that the sampling variance of T is minimum subject to the fixed cost say c_0.

This is to say, we require to minimize

$$np(1 - p) = Q(say) \quad (9)$$

Subject to:

$$\frac{N\pi}{\lambda}\left[1 - \left\{1 - p(1 - e^{-\lambda})\right\}^n\right] = c_0 \quad (10)$$

Equation (9) relates to minimization of the variance subject to the budget constraint where Q is taken as variance.

The budget constraint relation (10):

$$=> 1 - \left\{1 - p(1 - e^{-\lambda})\right\}^n = \frac{\lambda c_0}{N\pi}$$

$$=> n log\left[1 - p\left(1 - e^{-\lambda}\right)\right] = log\left(1 - \frac{\lambda c_0}{N\pi}\right)$$

$$=> n = \frac{log\left(1 - \frac{\lambda c_0}{N\pi}\right)}{log\left[1 - p\left(1 - e^{-\lambda}\right)\right]} \quad (11)$$

Substituting the value of n from (11) is the objective function (9), we get,

$$Q = \frac{log\left(1-\frac{\lambda c_0}{N\pi}\right)}{log\left[1-p\left(1-e^{-\lambda}\right)\right]} p(1-p) \qquad (12)$$

$$\frac{dQ}{dp} = \frac{log\left(1-\frac{\lambda c_0}{N\pi}\right).\,log\left[1-p\left(1-e^{-\lambda}\right)\right](1-2p) + \frac{p(1-p)\left(1-e^{-\lambda}\right)}{1-p\left(1-e^{-\lambda}\right)}}{\left[log\left\{1-p\left(1-e^{-\lambda}\right)\right\}\right]^n}$$

Putting $\frac{dQ}{dp}=0$, we get:

$$\left\{1-p\left(1-e^{-\lambda}\right)\right\}(1-2p)log\left\{1-p\left(1-e^{-\lambda}\right)\right\} + p(1-p)\left(1-e^{-\lambda}\right)=0 \qquad (13)$$

Further substituting $1-p\left(1-e^{-\lambda}\right)=y$, (13) reduces in

$$y^2\left[2logy-1\right]+y\left\{\left(1-e^{-\lambda}\right)(1-logy\right\}-e^{-\lambda}=0 \qquad (14)$$

For , $y = 1$ the quadratic equation has the form:

$$-y^2+y\left(1-e^{-\lambda}\right)-e^{-\lambda}=0$$

It can be easily seen that the above equation has two roots as $y = 1$ and $y = e^{-\lambda}$. The first corresponds to uniform type of care while the second corresponds to the maximum period on the average.

Substituting the values of y as 1 and $e^{-\lambda}$ in (13), we get $p = 0$ and $p = 1$, respectively.

However, if π, λ, c_0 and N are known, then the solution is obtainable by successive approximation and resubstituting the value of y, we get the estimate of p, which further gives the estimate of n from (11).

It may be noted that Equation (14) has got two roots as $y = 1$ and $y = e^{-\lambda}$. Both signify the type of care. The first corresponds to uniform type of care, while the second corresponds to the maximum period on the average. Substituting the values of y as 1 and $e^{-\lambda}$ in (13), we get p=0 and p=1, respectively.

Medical care for a maximum period on the average

In some of the situations, it may not be practicable to reduce the variation of the distribution of T and another alternative optimization scheme would be to provide the medical care for a maximum period, on the average, of course, subject to the budget constraint of the hospitals.

In other words, we require to maximize:

$$np = z(say) \qquad (15)$$

Subject to:

$$\frac{N\pi}{\lambda}\left[1-\left\{1-p(1-e^{-\lambda})\right\}^n\right]=c_0$$

In (15), Z is taken as the mean/average of the distribution under budget constraint.

The solution of the above constraint for n is given in (11); substituting the value of n in (15), we get:

$$z = \frac{log\left(1-\frac{\lambda c_0}{N\pi}\right)}{log\left[1-p\left(1-e^{-\lambda}\right)\right]}.\,p \qquad (16)$$

From (16),

$$\frac{dz}{dp} = \frac{log\left(1-\frac{\lambda\, c_0}{N\pi}\right).\,log\left[1-p\left(1-e^{-\lambda}\right)\right]+\frac{p\left(1-e^{-\lambda}\right)}{1-p\left(1-e^{-\lambda}\right)}}{\left[log\left\{1-p\left(1-e^{-\lambda}\right)\right\}\right]^2} \qquad (17)$$

Putting $\frac{dz}{dp}=0$, we get:

$$\left\{1-p\left(1-e^{-\lambda}\right)\right\}log\left[1-p\left(1-e^{-\lambda}\right)\right]+p\left(1-e^{-\lambda}\right)=0 \qquad (18)$$

Further substituting $1-p\left(1-e^{-\lambda}\right)=y$ in (18), we get:

$$ylogy+p(1-y)=0$$

The above equation has the only root $y = 1$ (in the interval $0 \le y \le 1$) which again implies $p=0,\ as\ e^{-\lambda}=1$ is inadmissible. This precisely provides zero as the average medical care duration is minimum.

However, the maximum average duration of medical care subject to a fixed hospital budget is realizable if the value of n is fixed to a minimum say $n = n_0$ (the ceiling may be decided by the hospital authorities in appropriate situations).

This implies that the budget constraint is:

$$\frac{N\pi}{\lambda}\left[1-\left\{1-p(1-e^{-\lambda})\right\}^{n_0}\right]=c_0$$

or

$$1-\left\{1-p(1-e^{-\lambda})\right\}^{n_0}=\frac{\lambda c_0}{N\pi}$$

$$=> 1-\left[\begin{array}{c}1-n_0.p\left(1-e^{-\lambda}\right)+C\left(n_0,2\right)p^2\left(1-e^{-\lambda}\right)^2 \\ +\ldots+\left(-1\right)^{n_0}p^{n_0}\left(1-e^{-\lambda}\right)^{n_0}\end{array}\right]=\frac{\lambda c_0}{N\pi}$$

which is a polynomial of degree n_0 in p. The maximum value of p $(0 \le p \le 1)$ say $\hat{p}$ corresponds to the optimal mean duration of medical care $n_0\,\hat{p}$.

The model can be extended further for situations commonly occurring in the case of surgical operations where the patient must be admitted at least 1 day before the date of surgery, for certain routine clinical tests. Thus, in this situation, the random variable T follows a truncated binomial distribution, in which the probability at $T = 0$ is truncated.

Now, the average duration of time per patient in this case:

$$= \frac{1}{1-(1-p)^n} \sum_{r=0}^{n} \frac{(1-e^{-\lambda r})}{\lambda} \; C(n,r)\, p^r (1-p)^r$$

$$= \frac{1}{1-(1-p)^n} \frac{1}{\lambda}\left[1-\left\{1-p(1-e^{-\lambda})\right\}^n\right]$$

Hence, the total cost for patients:

$$= \frac{N\pi}{\left[1-(1-p)^{n_0}\right]} \frac{1}{\lambda}\left[1-\left\{1-p(1-e^{-\lambda})\right\}^{n_0}\right] \quad (19)$$

Further, let c_0 be the fixed budget of the hospital and be the maximum feasible value of n (to be decided by the hospital authorities in different situations).

Then, the budget constraint gives:

$$\frac{N\pi}{\lambda\left[1-(1-p)^{n_0}\right]}\left[1-\left\{1-p(1-e^{-\lambda})\right\}^{n_0}\right] = c_0$$

Or

$$1-\left\{1-p\left(1-e^{-\lambda}\right)\right\}^{n_0} = \frac{\lambda c_0\left[1-(1-p)^{n_0}\right]}{N\pi}$$

The above result on further simplification reduces to a polynomial of degree n_0 in p. The maximum root of $\tilde{p}$ say p $(0 \le \tilde{p} \le 1)$ will therefore correspond to the optimal mean duration $n_0 p$.

Conclusion

Two types of medical care have been taken into consideration subject to the condition the patient must be admitted at least a day before the date of surgery, for certain routine clinical tests: one is the uniform type of medical care and another is the best type of medical care within the budget constraint. In this article, a model has been constructed for the best type of medical care subject to the available budget constraint in the hospital as well as their service time with an objective of obtaining distribution of T for providing a uniform type of medical care as far as practicable as well. $P = 1$ corresponds to the best type of care. These results are helpful in optimization of the level of hospitalization.

Financial support and sponsorship

Nil.

Conflicts of interest

There are no conflicts of interest.

References

1. Afrane S, Appah A. Queuing theory and the management of waiting-time in hospitals: The case of Angio Gold Ashanti Hospital in Ghana. Int J Acad Res Bus Soc Sci 2014;4:34-44.
2. Biswas S. On the extension of some results of counter models with Poisson inputs and their applications. J Indian Stat Assoc 1981;10:45-53.
3. Garwood F. An application of the theory of probability to the operation of vehicular controlled traffic signals. J R Stat Soc1940;7 Suppl B: 65-77.
4. Oliver RM. Distribution of gaps and Blocks in a traffic stream. Oper Res 1962;10:197-217.
5. Adele M, Barry S. Modeling Patient Flow in Hospital Using Queuing Theory. Unpublished Manuscript; 2005.
6. Cochran KJ, Bharti A. A multi-stage stochastic methodology for whole hospital bed planning under peak loading. Int J Ind Syst Eng 2006;1:8-35.
7. Davies R, Davies HT. Modelling Patient Flows and Resource Provisions in Health Systems. Omega-Oxford Pergamon Press; 2005;22:123-31.
8. Green LV. Queuing analysis in healthcare. In: Hall RW, editor. Patient Flow: Reducing Delay in Healthcare Deliver. New York: Springer; 2006a.
9. Green LV, Soares J, Giglio JF, Green RA. Using queuing theory to increase the effectiveness of emergency department provider staffing. Acad Emerg Med 2006;13:61-8.
10. Hall R, Belson D, Murali P, Dessouky M. Modeling patient flows through the health care system. In: Hall RW, editor. Patient Flow: Reducing Delay in Healthcare Delivery. New York: Springer; 2006. p. 1-44.
11. Ozcan YA. Quantitative Methods in Health Care Management: Techniques and applications 1st ed. Jossey-Bass Publications; 2006.
12. Roche KT, Cochran JK, Fulton IA. Improving Patient Safety by Maximizing Fast-Track Benefits in the Emergency Department – A Queuing Network Approach, in Proceedings of the 2007. Industrial Engineering Research Conference; 2007. p. 619-24.
13. Biswas S, Nautiyal BL, Tyagi RN. Some results of a stochastic process associated with Geiger Muller (G.M.) counter model. Sankhya 1983;45:271-83.
14. Singh V. Use of Queuing Models in Health Care Decision Analysis. Department of Health Policy and Management, University of Arkansas for Medical Science; 2006.
15. Raff MS. Distribution of blocks in an uncongested stream of Automobile traffic. J Am Stat Assoc 1951;46:114-23.
16. Karlin S, Taylor H. A First Course in Stochastic Process. 2nd ed. New York, San Francisco: Academic Press; 1975.